A Direct Least-Squares (DLS) Method for PnP

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Abstract

We present a Direct Least-Squares (DLS) method for computing all of the solutions of the perspective-n-point camera pose determination problem (PnP) in the general case ($n \geq 3$). We directly compute *all* minima of a nonlinear least-squares cost function, without relying on an initial guess or iterative techniques. We manipulate the cost function into polynomial form, and note that its optimality conditions comprise a system of three 3rd order polynomial equations. Subsequently, we utilize the Multiplication Matrix to compute all roots of the system directly, and hence, all local minima of the cost function.

Contributions

- Direct least-squares solution of PnP, for $n \geq 3$
- Complexity only linear in the number of points

Problem Description

- Perspective-n-point pose determination problem:

Given: observations of n points, ${}^S\bar{\mathbf{r}}_1, \ldots, {}^S\bar{\mathbf{r}}_n$, in one image, whose global-frame coordinates ${}^G\mathbf{r}_1, \ldots, {}^G\mathbf{r}_n$ are known

Compute: six degrees-of-freedom transformation $\{{}^S_G\mathbf{C}, {}^S\mathbf{p}_G\}$ from the Global frame {G} to the Camera (sensor) frame {S}

Assumptions: Known intrinsic calibration and correct data association

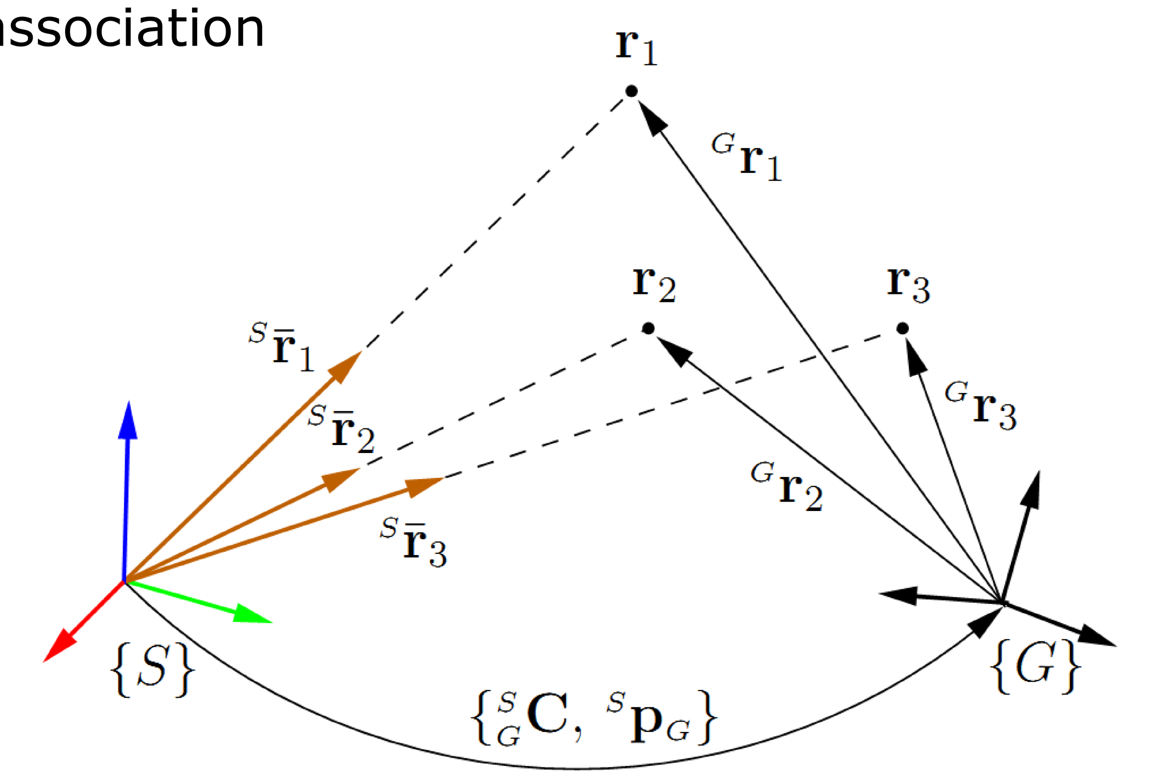


Fig. 1: Minimal case (n = 3) up to 4 possible solutions

Measurement Model

- Spherical camera model

$$\mathbf{z}_i = {}^S\bar{\mathbf{r}}_i + \boldsymbol{\eta}_i \qquad i = 1, \ldots, n$$
$${}^S\mathbf{r}_i = {}^S_G\mathbf{C}\,{}^G\mathbf{r}_i + {}^S\mathbf{p}_G$$

Nonlinear Least-Squares Cost Function

- Optimal position and orientation (pose) minimizes the following constrained cost function

$$\{{}^S_G\mathbf{C}^*, {}^S\mathbf{p}_G^*\} = \arg\min J$$

$$\text{subject to} \quad {}^S_G\mathbf{C}^T\,{}^S_G\mathbf{C} = \mathbf{I}_3, \quad \det\left({}^S_G\mathbf{C}\right) = 1$$

$$\alpha_i = ||{}^S_G\mathbf{C}\,{}^G\mathbf{r}_i + {}^S\mathbf{p}_G||$$

$$\text{where} \quad J = \sum_{i=1}^{n} ||\mathbf{z}_i - {}^S\bar{\mathbf{r}}_i||^2 \qquad (1)$$

$$= \sum_{i=1}^{n} ||\mathbf{z}_i - \frac{1}{\alpha_i}\left({}^S_G\mathbf{C}\,{}^G\mathbf{r}_i + {}^S\mathbf{p}_G\right)||^2$$

- Challenges: constraints, nonlinear, nonconvex, and multiple local minima!

DLS for Computing All Solutions

1. Transform measurement model
   - Exploit the geometric constraint relationships:

   $$\alpha_i\,{}^S\bar{\mathbf{r}}_i = {}^S_G\mathbf{C}\,{}^G\mathbf{r}_i + {}^S\mathbf{p}_G, \qquad i = 1, \ldots, n$$

   to express scale and translation as (see paper):

   $$\alpha_i = f\left({}^S_G\mathbf{C}, {}^S\mathbf{p}_G, {}^G\mathbf{r}_1, \ldots, {}^G\mathbf{r}_n, {}^S\bar{\mathbf{r}}_1, \ldots, {}^S\bar{\mathbf{r}}_n\right)$$

   $${}^S\mathbf{p}_G = g\left({}^S_G\mathbf{C}, {}^G\mathbf{r}_1, \ldots, {}^G\mathbf{r}_n, {}^S\bar{\mathbf{r}}_1, \ldots, {}^S\bar{\mathbf{r}}_n\right)$$

   - Substitute into (1) to obtain a cost function whose only unknown is the rotation ${}^S_G\mathbf{C}$
2. Represent ${}^S_G\mathbf{C}$ using Cayley-Gibbs-Rodriguez (CGR) rotation parameters: s_1, s_2, s_3
3. Convert cost function into a 4th order polynomial in three unknowns (CGR parameters): $J\left(s_1, s_2, s_3\right)$
4. Corresponding optimality conditions form a system of three 3rd order polynomial equations

   $$\nabla_{s_j} J\left(s_1, s_2, s_3\right) = F_j = 0, \qquad j = 1, 2, 3$$

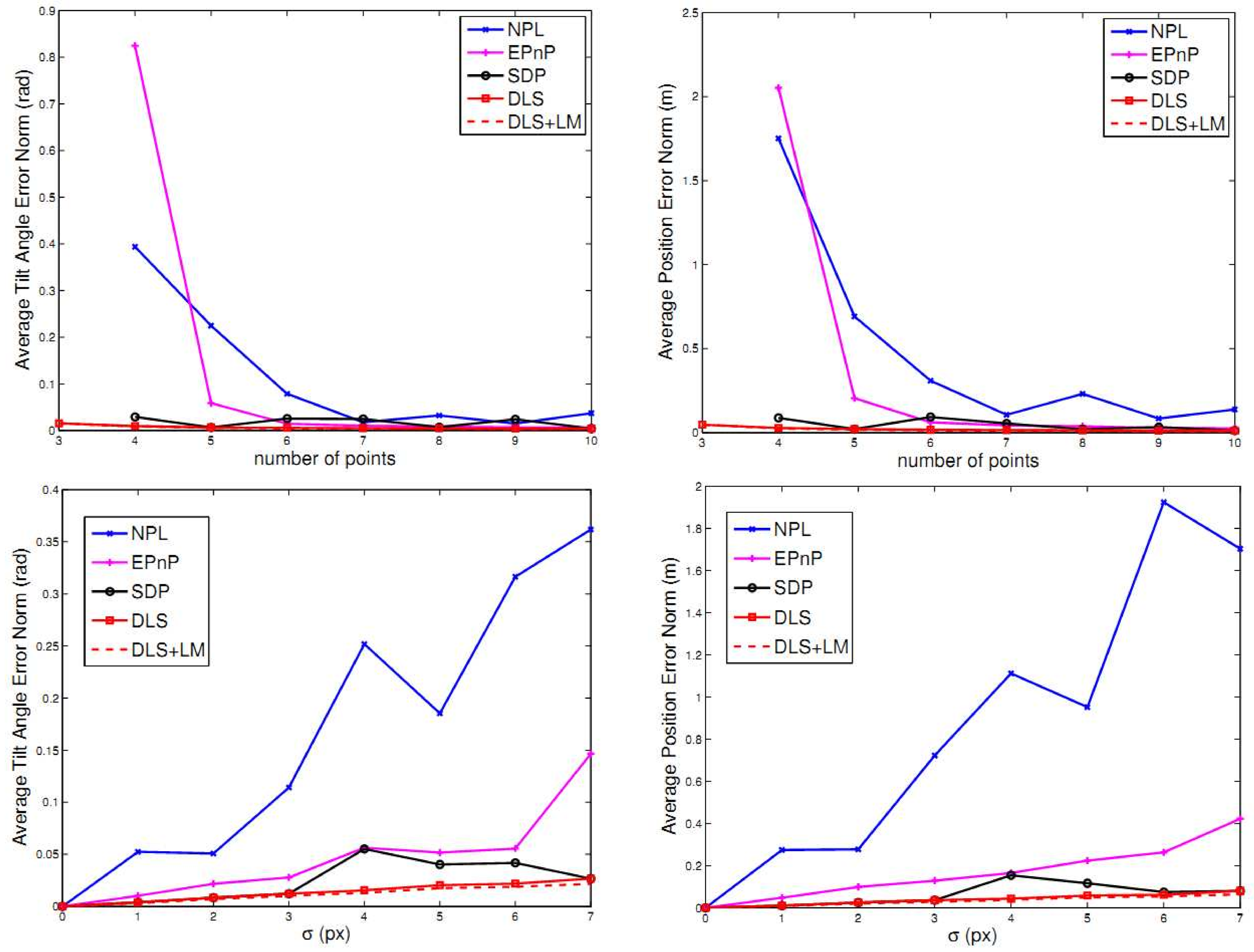
5. Solve system, $F_j = 0,\ j = 1, 2, 3$, using Multiplication Matrix (Eigen-decomposition of 27 x 27 matrix)

Key Results:

- Obtains all minima directly, we select global optimum by evaluating original cost function at all solutions
- Forming $F_j = 0,\ j = 1, 2, 3$ is linear in the # of points
- LS formulation is generic, and independent of number of points and scene layout

Simulations and Experimental Results

- Accuracy vs. number of points and pixel noise (σ) (average error computed over 100 Monte-Carlo trials)

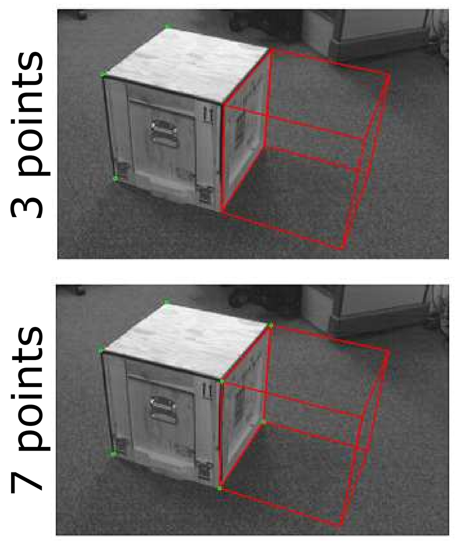


- **NPL:** Ansar & Daniilidis, "Linear pose estimation from points or lines" PAMI '03
- **EPnP:** Lepetit et al., "EPnP: An accurate O(n) solution to the PnP problem" IJCV '08
- **SDP:** Schweighofer et al., "Globally optimal O(n) solution to the PnP problem for general comera models", In Proc. of the Brittish Machine Vision Conf. '08
- **DLS:** Proposed Direct Least-Squares approach
- **DLS+LM:** Levenberg-Marquardt iterative minimization of original cost function, initialized with DLS (benchmark)

- Experimental results (PnP + virtual box reprojection)

3 points

7 points



n-points	Ori. Error Norm (rad)	Pos. Error Norm (m)
NPL4	2.87×10^{-3}	8.67×10^{-3}
NPL7	2.12×10^{-3}	2.42×10^{-3}
EPnP4	2.49×10^{-2}	2.33×10^{-2}
EPnP7	1.24×10^{-2}	3.41×10^{-3}
SDP4	4.26×10^{-3}	9.82×10^{-3}
SDP7	3.86×10^{-4}	3.49×10^{-4}
DLS3	5.41×10^{-3}	1.02×10^{-2}
DLS4	4.28×10^{-3}	9.83×10^{-3}
DLS7	4.29×10^{-4}	3.35×10^{-4}

Conclusions and Future Work

- Accuracy comparable with Maximum Likelihood Estimate
- Applicable in general scenarios (of $n \geq 3$ points) with planar or non-planar scenes
- On-going work to deal with unknown data association and presence of outliers

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